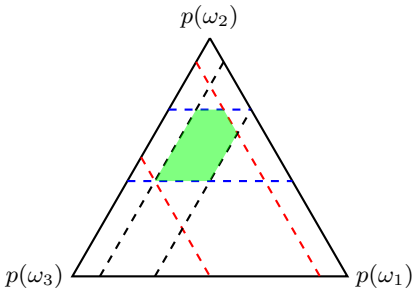
Exercises for the 8th SIPTA Summer School

July 24th - July 28th, 2018

1 Exercises

1. Consider the probability/credal set described by the constraints

$$p(\omega_1) \in [0.1, 0.3], p(\omega_2) \in [0.4, 0.7], p(\omega_3) = [0.1, 0.5]$$

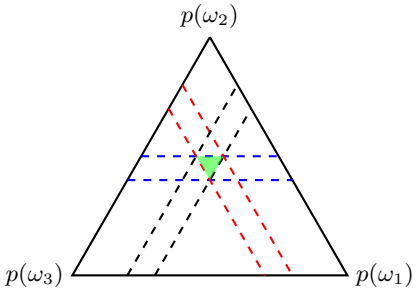


Show that these induce a belief function, e.g., by computing the lower probabilities and showing that the Möbius inverse is non-negative.

	$\{\omega_1\}$	$\{\omega_2\}$	$\{\omega_3\}$	$\{\omega_1, \omega_2\}$	$\{\omega_1, \omega_3\}$	$\{\omega_2, \omega_3\}$	Ω
$\underline{P}$							
m							

2. Consider the probability/credal set described by the constraints

$$p(\omega_1) \in [0.2, 0.3], p(\omega_2) \in [0.4, 0.5], p(\omega_3) = [0.2, 0.3]$$



Show that these do not induce a belief function, e.g., by computing the lower probabilities and showing that the Möbius inverse is negative for some set (hint: focus on big ones), or by showing that it is not 3-monotone.

	$\{\omega_1\}$	$\{\omega_2\}$	$\{\omega_3\}$	$\{\omega_1, \omega_2\}$	$\{\omega_1, \omega_3\}$	$\{\omega_2, \omega_3\}$	Ω
$\underline{P}$							
m							

3. Consider the space $\Omega = \{a, b, c\}$ and the following mass functions:

$$m_1(\{b\}) = 0.3, m_1(\{b, c\}) = 0.2, m_1(\{a, b, c\}) = 0.5$$

$$m_2(\{a\}) = 0.2, m_2(\{b\}) = 0.3, m_2(\{c\}) = 0.3, m_2(\{a, b, c\}) = 0.2$$

$$m_3(\{a, b\}) = 0.3, m_3(\{a, c\}) = 0.3, m_3(\{a\}) = 0.4$$

Build the partial order $\sqsubseteq$ between m_1, m_2, m_3, reminding that

$$m_i \sqsubseteq m_j \text{ iff } \underline{P}_i(A) \geq \underline{P}_j(A) \text{ for all } A$$

4. The hotel provides the following plates for breakfast

a=Century egg, b=Rice, c=Croissant, d=Raisin Muffin

In a survey about their choices, respondents gave the reply

$$m(\{a, b\}) = \alpha,\ m(\{c, d\}) = 1 - \alpha$$

We learn that customer C does not like eggs nor raisins ($C = \{b, c\}$), what can we tell about him choosing Rice by applying the focusing operation?

5. The hotel provides the following plates for breakfast

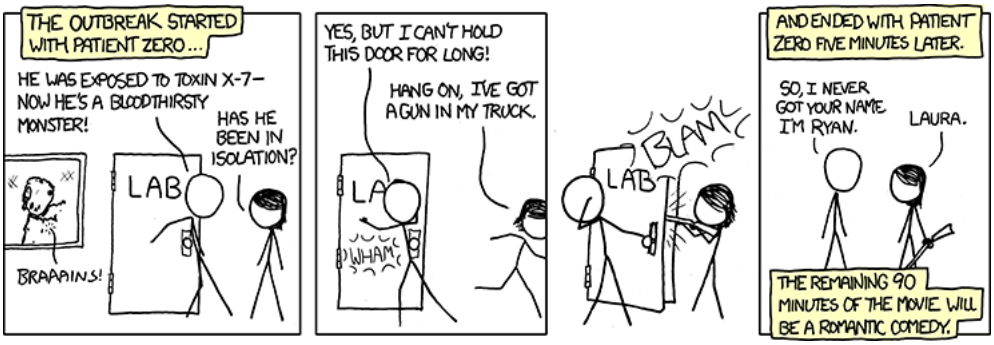
a=Century egg, b=Rice, c=Croissant, d=Raisin Muffin

In a survey about their choices, respondent gave the reply

$$m(\{a, b\}) = \alpha,\ m(\{c, d\}) = 1 - \alpha$$

We learn that suppliers no longer have eggs nor raisins ($C = \{b, c\}$), what is the proportion of rice we should buy to satisfy customers by applying the revision operation?

6. A zombie apocalypse has happened, and you must recognize possible threats/supports



The possibilities Ω are

- Zombie (Z)
- Friendly Human (F)
- Hostile Human (H)
- Neutral Human (N)

The sources S_i are

- Half-broken heat detector (S_1)
- Paranoid watch guy 1 (S_2)
- Half-broken Motion detector (S_3)
- Sleepy watch guy 2 (S_4)

Given this table of contour functions, a weighted average and a decision based on maximal plausibility

	$\hat{\omega}^1 = Z$				$\hat{\omega}^2 = H$				$\hat{\omega}^3 = F$			
	Z	F	H	N	Z	F	H	N	Z	F	H	N
S_1	1	$0,5$	$0,5$	$0,5$	1	$0,5$	$0,5$	$0,5$	$0,5$	1	1	1
S_2	1	$0,2$	$0,8$	$0,2$	0	$0,3$	1	$0,3$	0	$0,4$	1	$0,4$
S_3	1	$0,5$	$0,5$	$0,5$	$0,5$	$0,7$	$0,8$	$0,7$	1	$0,5$	$0,5$	$0,5$
S_4	1	1	1	1	$0,2$	$0,2$	1	$0,5$	$0,2$	1	$0,4$	$0,8$
$\mathbf{w}_1 = (0.5, 0.5, 0, 0)$												
$\mathbf{w}_2 = (0, 0, 0.5, 0.5)$												

Choose $h_{\mathbf{w}_1}$ or $h_{\mathbf{w}_2}$? Given the data, can we find a strictly better weight vector?