Approximations of coherent lower probabilities by 2-monotone (or completely monotone) capacities

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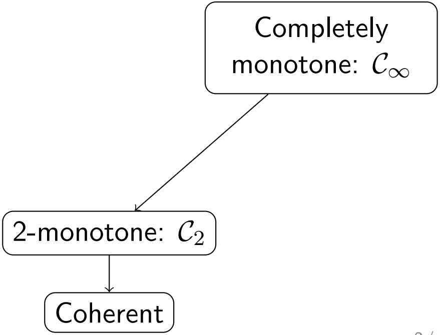
Models


Completely monotone: $\mathcal{C}_\infty$
2-monotone: $\mathcal{C}_2$
Coherent

Why (not) coherent lower probabilities?

- $\underline{P}$ is coherent $\iff$ $\underline{P} = \min \mathcal{M}(\underline{P})$. ✓
- In addition to this sensitivity analysis interpretation, coherence also has a **behavioural** interpretation. ✓
- However, the structure of $\mathcal{M}(\underline{P})$ may be complex. ✗
- The extension of $\underline{P}$ to gambles is not unique. ✗

Why (not) 2-monotonicity?

If $\underline{P}$ is 2-monotone:

- It has a unique extension to gambles (by means of the Choquet integral). ✓
- There is a simple procedure for computing the number of extreme points of $\mathcal{M}(\underline{P})$. ✓
- Most particular cases in the literature are 2-monotone anyway. ✓
- But the behavioural interpretation of 2-monotonicity is not too clear. ✗

Why (not) Completely monotonicity?

If $\underline{P}$ is completely monotone:

- Same advantages than 2-monotone ones. ✓
- Interpretation in terms of the Möbius inverse. ✓
- Necessity measureas and p-boxes are particular cases. ✓
- Belief functions are the key notion of Evidence Theory. ✓
- Sometimes too restrictive. ✗

Formulation of the problem

We look for a 2-monotone lower probability $\underline{Q}$ such that:

(a) $\underline{Q}$ does not include additional information: $\underline{Q} \leq \underline{P}$. It is called an **outer approximation**.

(b) $\underline{Q}$ is as *close* as possible. $\underline{Q}$ is **undominated** if there is no other 2-monotone $\underline{Q}'$ such that $\underline{Q} \lneq \underline{Q}' \leq \underline{P}$.

Approximation of coherent lower probabilities by 2-monotone measures. A. Bronevich, T. Augustin, ISIPTA 2009.

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Approximation of coherent lower probabilities by 2-monotone measures. A. Bronevich, T. Augustin, ISIPTA 2009.

We consider the distance proposed by Baroni and Vicig:

$$d(\underline{P}, \underline{Q}) = \sum_{E \subseteq \mathcal{X}} (\underline{P}(E) - \underline{Q}(E)).$$

An uncertainty interchange format with imprecise probabilities. P. Baroni, P. Vicig, IJAR 2015.

Overview

OA via linear programming

We look for a 2-monotone lower probability $\underline{Q}, m_{\underline{Q}}$, such that $\underline{Q} \leq \underline{P}$.

$$\min \sum_{E \subseteq \mathcal{X}} \left(\underline{P}(E) - \sum_{B \subseteq E} m_{\underline{Q}}(B) \right) \tag{LP-2monot}$$

subject to:

$$\sum_{B \subseteq \mathcal{X}} m_{\underline{Q}}(B) = 1, \qquad m_{\underline{Q}}(\emptyset) = \emptyset. \tag{LP-2monot.1}$$

$$\sum_{\{x_i, x_j\} \subseteq A \subseteq E} m_{\underline{Q}}(A) \geq 0, \quad \forall E \subseteq \mathcal{X},\ \forall x_i, x_j \in E,\ x_i \neq x_j. \tag{LP-2monot.2}$$

$$m_{\underline{Q}}(\{x_i\}) \geq 0, \quad \forall x_i \in \mathcal{X}. \tag{LP-2monot.3}$$

$$\sum_{B \subseteq E} m_{\underline{Q}}(B) \leq \underline{P}(E) \quad \forall E \subseteq \mathcal{X}. \tag{LP-2monot.4}$$

Some characterizations of lower probabilities and other monotone capacities through the use of Möbius inversion. A. Chateauneuf, J. Jaffray, MSS 1989.

OA via linear programming

- The feasible region of (LP-2monot) is non-empty: it includes the vacuous lower probability $(m(\mathcal{X}) = 1)$.
- The optimization problem (LP-2monot) has a (possibly infinitely) solution(s).
- Any optimal solution is an undominated OA in $\mathcal{C}_2$.

OA via linear programming

- The feasible region of (LP-2monot) is non-empty: it includes the vacuous lower probability ($m(\mathcal{X}) = 1$).
- The optimization problem (LP-2monot) has a (possibly infinitely) solution(s).
- Any optimal solution is an undominated OA in $\mathcal{C}_2$.
- However, there may be no solution attaining the value $\underline{P}(A)$ for a given A.
- If for a fixed A we add the restriction

$$\sum_{B \subseteq A} m_{\underline{Q}}(B) = \underline{P}(A) \qquad \text{(LP-2monot.5)}$$

then any optimal solution of (LP-2monot) subject to (LP-2monot.1)–(LP-2monot.5) is an undominated OA in $\mathcal{C}_2$ satisfying $\underline{Q}(A) = \underline{P}(A)$.

Further properties

- If we denote by $(\underline{Q}_i)_{i \in I}$ the optimal solutions of (LP-2monot) subject to (LP-2monot.1)–(LP-2monot.4) or (LP-2monot.1)–(LP-2monot.5) for a fixed A, then

$$\underline{P}(E) = \max_{i \in I} \underline{Q}_i(E) \quad \forall E \subseteq \mathcal{X}.$$

- If $\underline{Q}$ is an undominated OA of $\underline{P}$ in $\mathcal{C}_2$, then $\underline{Q}(\{x\}) = \underline{P}(\{x\})$ and $\overline{Q}(\{x\}) = \overline{P}(\{x\})$ for every x.
- There are undominated OA in $\mathcal{C}_2$ that cannot be obtained via linear programming.
- Open: is the set of undominated solutions convex?

Overview

OA via linear programming

We look for a belief function Bel, with Möbius inverse m, such that $Bel \leq \underline{P}$.

$$\min \sum_{E \subseteq \mathcal{X}} \Big(\underline{P}(E) - \sum_{B \subseteq E} m(B) \Big) \qquad \text{(LP-Bel)}$$

subject to:

$$\sum_{B \subseteq \mathcal{X}} m(B) = 1, \qquad m(B) \geq 0 \quad \forall B \subseteq \mathcal{X}. \qquad \text{(LP-Bel.1)}$$

$$\sum_{B \subseteq E} m(B) \leq \underline{P}(E) \quad \forall E \subseteq \mathcal{X}. \qquad \text{(LP-Bel.2)}$$

Generation, combination and extension of random set approximations to coherent lower and upper probabilities. J.W. Hall, J. Lawry, RESS 2004.

Completely monotone outer approximations of lower probabilities on finite possibility spaces. E.Quaeghebeur, Springer 2011.

OA via linear programming

- The feasible region of (LP-Bel) is non-empty: it includes the simple support function ($m(A) = \underline{P}(A)$ and $m(\mathcal{X}) = 1 - \underline{P}(A)$).
- The optimization problem (LP-Bel) has a (possibly infinitely) solution(s).
- Any optimal solution is an undominated OA in $\mathcal{C}_\infty$.

OA via linear programming

- The feasible region of (LP-Bel) is non-empty: it includes the simple support function ($m(A) = \underline{P}(A)$ and $m(\mathcal{X}) = 1 - \underline{P}(A)$).
- The optimization problem (LP-Bel) has a (possibly infinitely) solution(s).
- Any optimal solution is an undominated OA in $\mathcal{C}_\infty$.
- However, there may be no solution attaining the value $\underline{P}(A)$ for a given A.
- If for a fixed A we add the restriction

$$\sum_{B \subseteq A} m(B) = \underline{P}(A) \qquad \text{(LP-Bel.A)}$$

then any optimal solution of (LP-Bel) subject to (LP-Bel.1)–(LP-Bel.3) is an undominated OA in $\mathcal{C}_\infty$ satisfying $Bel(A) = \underline{P}(A)$.

Further properties

- If we denote by $(Bel_i)_{i\in I}$ the optimal solutions of (LP-Bel) subject to (LP-Bel.1)–(LP-Bel.2) or (LP-Bel.1)–(LP-Bel.3) for a fixed A, then

$$\underline{P}(E) = \max_{i\in I} Bel_i(E) \quad \forall E \subseteq \mathcal{X}.$$

- If Bel is an undominated OA of $\underline{P}$ in $\mathcal{C}_\infty$, it may not happen that $Bel(\{x\}) = \underline{P}(\{x\})$ and $Pl(\{x\}) = \overline{P}(\{x\})$.
- There are undominated OA in $\mathcal{C}_\infty$ that cannot be obtained via linear programming.
- Open: is the set of undominated solutions convex?

Overview

Quadratic programming

Instead of the distance of Baroni and Vicig, we may consider the **quadratic distance**:

$$\tilde{d}(\underline{P}, \underline{Q}) := \sum_{E \subseteq \mathcal{X}} (\underline{P}(E) - \underline{Q}(E))^2.$$

- The optimization problem has a unique solution in $\mathcal{C}_2$ and $\mathcal{C}_\infty$, unlike the linear programming ones.
- This solution is an undominated OA in $\mathcal{C}_2$ or $\mathcal{C}_\infty$, and it may not be a solution of any of the linear programming problems.
- Interpretation?

Total variation distance

Given two probability measures P_1 and P_2, their **total variation** is defined as

$$||P_1 - P_2|| = \max_{E \subseteq \mathcal{X}} |P_1(E) - P_2(E)|.$$

This definition can be equivalently expressed as:

$$||P_1 - P_2|| = \frac{1}{2} \sum_{x \in \mathcal{X}} |P_1(\{x\}) - P_2(\{x\})|.$$

This can be extended to coherent lower probabilities in a number of (not necessarily equivalent) ways:

$$d_1(\underline{P}_1, \underline{P}_2) = \max_{E \subseteq \mathcal{X}} |\underline{P}_1(E) - \underline{P}_2(E)|,$$

$$d_2(\underline{P}_1, \underline{P}_2) = \frac{1}{2} \sum_{x \in \mathcal{X}} |\underline{P}_1(\{x\}) - \underline{P}_2(\{x\})|,$$

$$d_3(\underline{P}_1, \underline{P}_2) = \sup_{P_1 \in \mathcal{M}(\underline{P}_1), P_2 \in \mathcal{M}(\underline{P}_2)} ||P_1 - P_2||.$$

OA via total variation

Thus, we may consider the optimization problem of obtaining an OA of $\underline{P}$ in $\mathcal{C}_2$ or $\mathcal{C}_\infty$ that minimizes one of these distances.

- None of the distances d_1, d_2, d_3 guarantees a unique solution.
- Moreover, the solutions to the problem may not be undominated!

The Weber set

For any permutation σ of $\{1,\ldots,n\}$, define P_σ by

$$P_\sigma(\{x_{\sigma(1)},\ldots,x_{\sigma(k)}\}) = \underline{P}(\{x_{\sigma(1)},\ldots,x_{\sigma(k)}\}) \ \forall k.$$

If S_n denotes the set of permutations of $\{1,\ldots,n\}$, then

$$W(\underline{P}) = \{P_\sigma \mid \sigma \in S_n\}$$

is called the **Weber set** associated with $\underline{P}$.

If $\underline{P}$ is 2-monotone, then $ext(\mathcal{M}(\underline{P})) = W(\underline{P})$.

Set functions, games and capacities in decision making. M. Grabisch, Springer 2016.

OA via the Weber set

When $\underline{P}$ is a coherent lower probability, $\mathcal{M}(\underline{P}) \subseteq conv(W(\underline{P}))$, and $\mathcal{M}(\underline{P}) = conv(W(\underline{P})) \iff \underline{P}$ 2-monotone.

Supermodularity: Applications to convex games and to the greedy algorithm for LP. T. Ichiishi, JET 1981.

Thus, we may use the lower envelope of $W(\underline{P})$ to outer approximate $\underline{P}$:

$$\underline{Q} = \min conv(W(\underline{P})).$$

- This lower envelope may not be 2-monotone for $n > 4$.
- Even if it is 2-monotone, it may not be an undominated OA.

Iterative (minimal) rescaling method

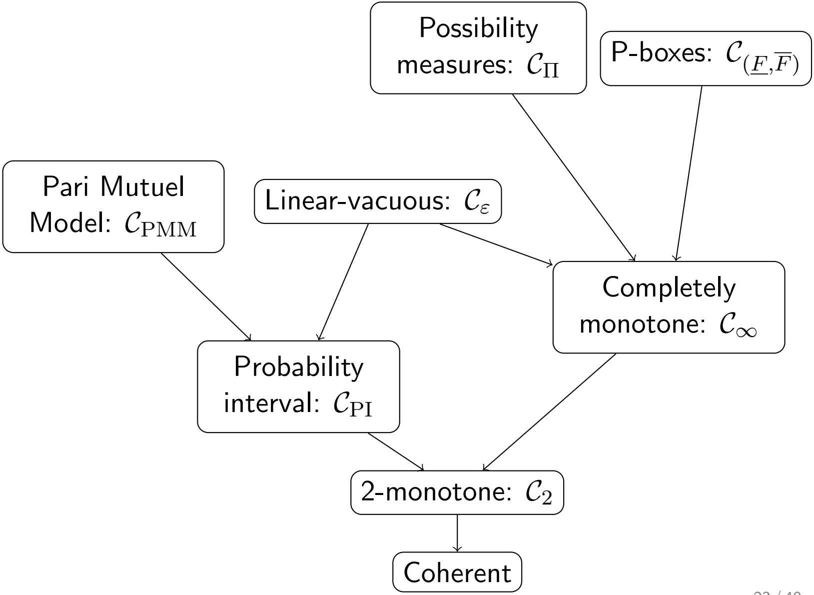
- Hall & Lawry: Iterative Rescaling Method.
- Quaeghebeur: Iterative Minimal Rescaling Method.
- Both methods may give non undominated OA.

Generation, combination and extension of random set approximations to coherent lower and upper probabilities. J.W. Hall, J. Lawry, RESS 2004.

Completely monotone outer approximations of lower probabilities on finite possibility spaces. E.Quaeghebeur, Springer 2011.

Overview

Particular cases



OA using probability intervals

Let $\underline{P}$ be a coherent lower probability with conjugate upper probability $\overline{P}$. Define the probability interval $\mathcal{I}$ by:

$$\mathcal{I} = \{[l_i, u_i] = [\underline{P}(\{x_i\}), \overline{P}(\{x_i\})] \mid i = 1, \dots, n\},$$

and denote by l, u the lower and upper probability it induces.

- $\mathcal{I}$ is a coherent probability interval.
- l is the **unique** undominated OA of $\underline{P}$ in $\mathcal{C}_{\mathrm{PI}}$.

Pari-mutuel models

The **Pari Mutuel Model** (PMM) is a betting scheme originated in horse racing. It is determined by a probability measure P_0 and a distortion factor $\delta > 0$.
They determine a coherent lower probability by:

$$\underline{P}(A) = \max\{0, (1+\delta)P_0(A) - \delta\} \ \forall A \subseteq \mathcal{X}.$$

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Define the constant value $\delta > 0$ and the probability P_0 by:

$$\delta = \sum_{i=1}^{n} \overline{P}(\{x_i\}) - 1, \quad P_0(\{x_i\}) = \frac{\overline{P}(\{x_i\})}{1+\delta} \ \forall i.$$

Let $\underline{Q}_\delta$ be the lower probability associated with the PMM (P_0, δ).

- $\underline{Q}_\delta$ is the **unique** undominated OA of $\underline{P}$ in $\mathcal{C}_{\mathrm{PMM}}$.

Linear-vacuous mixtures

Given a probability measure P_0 and $\varepsilon \in (0,1)$, we define the ε-**contamination model** by

$$\underline{P}(A) = \begin{cases} (1-\varepsilon)P_0(A) & \text{if } A \neq \mathcal{X}. \\ 1 & \text{if } A = \mathcal{X}. \end{cases}$$

Linear-vacuous mixtures

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Assume $\underline{P}$ satisfies $\sum_{j=1}^{n} \underline{P}(\{x_j\}) > 0$ and define:

$$\varepsilon = 1 - \sum_{j=1}^{n} \underline{P}(\{x_j\}), \quad P_0(\{x_i\}) = \frac{\underline{P}(\{x_i\})}{\sum_{j=1}^{n} \underline{P}(\{x_j\})} \ \forall i.$$

Let $\underline{P}_\varepsilon$ be the ε-contamination model they determine.

- $\underline{P}_\varepsilon$ is the **unique** undominated OA of $\underline{P}$ in $\mathcal{C}_\varepsilon$.
- If $\sum_{j=1}^{n} \underline{P}(\{x_j\}) = 0$, then $\underline{P}$ has no OA in $\mathcal{C}_\epsilon$.

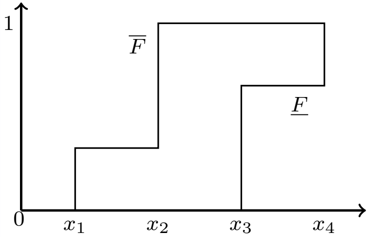
OA using p-boxes

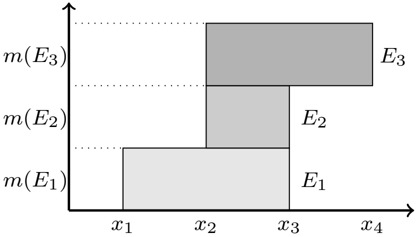
P-box: belief function whose focal sets are ordered intervals:

Interval: If E is focal, $E = \{x \in \mathcal{X} \mid \min E \leq x \leq \max E\}$.

Ordered: If E_1, E_2 are focals, either

$$\min E_1 \leq \min E_2 \text{ and } \max E_1 \leq \max E_1 \text{ or}$$
$$\min E_2 \leq \min E_1 \text{ and } \max E_2 \leq \max E_1.$$





OA using p-boxes: $\mathcal{X}$ ordered

2-monotone OA

Completely monotone OA

Other approximations

Particular cases

OA of coherent lower previsions

Inner approximations

Conclusions and Open Problems

Proposition

There is a unique undominated OA in $\mathcal{C}_{(\underline{F},\overline{F})}$, and it is given by:

$$\underline{F}(x_i) = \underline{P}(\{x_1, \dots, x_i\}), \qquad \overline{F}(x_i) = \overline{P}(\{x_1, \dots, x_i\}).$$

Then:

$$\underline{P}_{(\underline{F},\overline{F})}(A) = \min\{P(A) \mid \underline{F} \le F_P \le \overline{F}\}.$$

Probability boxes on totally preordered spaces for multivariate modelling. M. Troffaes, S. Destercke, IJAR 2011.

OA using p-boxes: $\mathcal{X}$ not ordered

Let σ be a permutation of $\{1, \ldots, n\}$ and $\leq_\sigma$ the order given by σ:

$$x_{\sigma(1)} \leq_\sigma \ldots \leq_\sigma x_{\sigma(n)}.$$

Proposition

1. *Define the* **generalised p-box** $(\underline{F}_\sigma, \overline{F}_\sigma)$ *by:*

$$\underline{F}_\sigma(x_{\sigma(i)}) = \underline{P}(\{x_{\sigma(1)}, \ldots, x_{\sigma(i)}\}),$$
$$\overline{F}_\sigma(x_{\sigma(i)}) = \overline{P}(\{x_{\sigma(1)}, \ldots, x_{\sigma(i)}\}).$$

   Then, $(\underline{F}_\sigma, \overline{F}_\sigma)$ *is a undominated OA in* $\mathcal{C}_{(\underline{F}, \overline{F})}$.

2. *All the undominated OA in* $\mathcal{C}_{(\underline{F}, \overline{F})}$ *are the p-boxes* $(\underline{F}_\sigma, \overline{F}_\sigma)$ *for any* σ.

OA using possibility measures

We look for a possibility Π such that $\overline{P} \leq \Pi$.

Theorem

Take σ a permutation of $\{1, \ldots, n\}$. We define Π by:

$\Pi(\{x_{\sigma(1)}\}) = \overline{P}(\{x_{\sigma(1)}\})$ *and*

$\Pi(\{x_{\sigma(i)}\}) = \max_{A \in \mathcal{A}_{\sigma(i)}} \overline{P}\left(A \cup \{x_{\sigma(i)}\}\right),$ *where*

$$\mathcal{A}_{\sigma(i)} = \left\{ A \subseteq \{x_{\sigma(1)}, \ldots, x_{\sigma(i-1)}\} \mid \overline{P}\left(A \cup \{x_{\sigma(i)}\}\right) > \max_{x \in A} \Pi(\{x\}) \right\},$$

and let $\Pi(A) = \max_{x \in A} \Pi(\{x\})$ for any other $A \subseteq \mathcal{X}$. Then:

1. *Π is a possibility measure.*
2. *Π is a undominated OA of $\overline{P}$.*
3. *All the undominated OA of $\overline{P}$ are of the form Π_σ.*

Dubois & Prade approach

For any permutation σ, define:

- $E_0^\sigma = \emptyset$.
- $E_j^\sigma = \{x_{\sigma(1)}, \ldots, x_{\sigma(j)}\}$ for $j = 1, \ldots, n$.
- $\pi_\sigma^{\mathrm{DP}}(x_{\sigma(j)}) = 1 - \underline{P}(E_{j-1}^\sigma)$ for $j = 1, \ldots, n$.
- $\Pi_\sigma^{\mathrm{DP}}(A) = \max_{x \in A} \pi_\sigma^{\mathrm{DP}}(x)$.

Fuzzy sets and statistical data. D. Dubois, H. Prade, EJOR 1986.

When upper probabilities are possibility measures. D. Dubois, H. Prade, FSS 1992.

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- $\Pi_\sigma^{\mathrm{DP}}(A) = \max_{x \in A} \pi_\sigma^{\mathrm{DP}}(x)$.

However...

- Π_σ^{DP} could be dominated!
- $\Pi_\sigma^{\mathrm{DP}} \geq \Pi_{\overline{\sigma}}$, where $\overline{\sigma} = (x_{\sigma(n)}, x_{\sigma(n-1)}, \dots, x_{\sigma(1)})$.

Fuzzy sets and statistical data. D. Dubois, H. Prade, EJOR 1986.

When upper probabilities are possibility measures. D. Dubois, H. Prade, FSS 1992.

Example

2-monotone OA
Completely monotone OA
Other approximations
Particular cases
OA of coherent lower previsions
Inner approximations
Conclusions and Open Problems

A	$\underline{P}(A)$	$\overline{P}(A)$
$\{x_1\}$	0.1	0.4
$\{x_2\}$	0	0.3
$\{x_3\}$	0	0.4
$\{x_4\}$	0.3	0.5
$\{x_1, x_2\}$	0.1	0.6
$\{x_1, x_3\}$	0.3	0.6
$\{x_1, x_4\}$	0.6	0.7
$\{x_2, x_3\}$	0.3	0.4
$\{x_2, x_4\}$	0.4	0.7
$\{x_3, x_4\}$	0.4	0.9
$\{x_1, x_2, x_3\}$	0.5	0.7
$\{x_1, x_2, x_4\}$	0.6	1
$\{x_1, x_3, x_4\}$	0.7	1
$\{x_2, x_3, x_4\}$	0.6	0.9
$\mathcal{X}$	1	1

Take $\sigma = (3, 2, 1, 4)$ and $\overline{\sigma} = (4, 1, 2, 3)$:

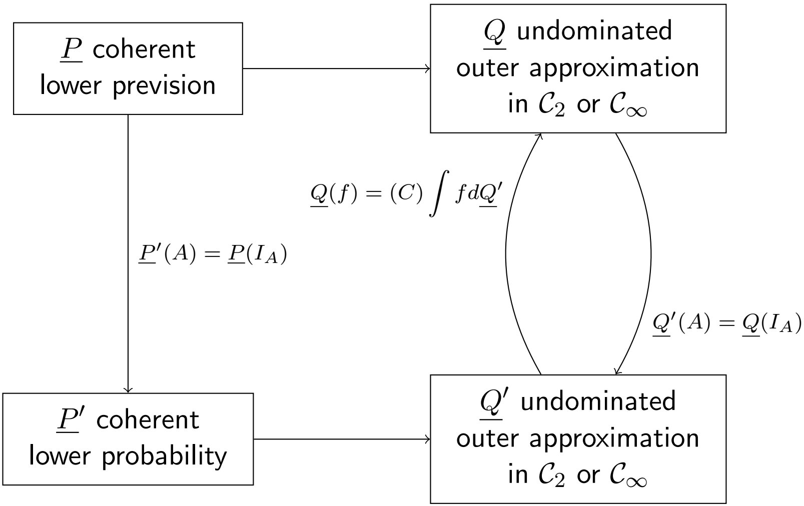
	Π_σ	$\Pi_{\overline{\sigma}}^{\mathrm{DP}}$
x_3	0.4	0.4
x_2	0.3	0.4
x_1	0.7	0.7
x_4	1	1

Overview

OA of coherent lower previsions

$$\begin{tikzcd}
\boxed{\text{$\underline{P}$ coherent lower prevision}} \arrow[r] \arrow[d, "\underline{P}'(A) = \underline{P}(I_A)"] & \boxed{\text{$\underline{Q}$ undominated outer approximation in $\mathcal{C}_2$ or $\mathcal{C}_\infty$}} \\
\boxed{\text{$\underline{P}'$ coherent lower probability}} \arrow[r] & \boxed{\text{$\underline{Q}'$ undominated outer approximation in $\mathcal{C}_2$ or $\mathcal{C}_\infty$}}
\end{tikzcd}$$

OA of coherent lower previsions



Overview

Inner approximations

- We can repeat the same arguments to look for inner approximations.
- Using a linear programming approach, we just need to substitute (LP-2monot.4) or (LP-bel.2) by:

$$\sum_{B \subseteq E} m_{\underline{Q}}(B) \geq \underline{P}(E) \quad \forall E \subseteq \mathcal{X}.$$

However...

- The existence of an inner approximation in $\mathcal{C}_\varepsilon$ and $\mathcal{C}_{\mathrm{PMM}}$ is not guaranteed, and when it exists, it may not be unique. The existence is neither guaranteed in $\mathcal{C}_\Pi$.
- An inner approximation adds new information to the model!!!

Overview

Summary and open problem

OA in the class $\mathcal{C}$	Unique optimal OA?	The optimal OAs retrieve $\underline{P}$?	Characterization of the optimal OA?	Coincide with $\underline{P}, \overline{P}$ on $\{x_i\}$?
$\mathcal{C}_2$	NO	YES	Open problem	YES
$\mathcal{C}_\infty$	NO	YES	Open problem	NO
$\mathcal{C}_{\mathrm{PI}}$	YES	NO	YES	YES
$\mathcal{C}_{\mathrm{PMM}}$	YES	NO	YES	Partially
$\mathcal{C}_\varepsilon$	YES	NO	YES	Partially
$\mathcal{C}_{(\underline{F},\overline{F})}$	YES	NO	YES	NO
$\mathcal{C}^*_{(\underline{F},\overline{F})}$	NO	YES	YES	NO
$\mathcal{C}_\Pi$	NO	YES	YES	NO

- Elicitation of an undominated OA?
- Using divergences instead of distances?

References

When upper probabilities are possibility measures. D. Dubois, H. Prade, FSS 1992.

An uncertainty interchange format with imprecise probabilities. P. Baroni, P. Vicig, IJAR 2005.

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Outer approximations of coherent lower probabilities using belief functions. I. Montes, E. Miranda, P. Vicig, SMPS-Belief Conference 2018.

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