$$X \hookrightarrow N(\mu,\sigma) \qquad H_0 \equiv (\mu,\sigma) = (\mu_0,\sigma_0) \qquad H_1 \equiv (\mu,\sigma) \neq (\mu_0,\sigma_0)$$

$$\begin{aligned}
L_0 &= \frac{1}{\sigma_0^n \sqrt{2\pi}^n} \cdot e^{-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\mu_0}{\sigma_0}\right)^2} \\
L &= \frac{1}{s^n \sqrt{2\pi}^n} \cdot e^{-\frac{1}{2}\frac{1}{s^2}\sum_{i=1}^{n}(x_i-\bar{x})^2} \\
&= \frac{1}{s^n \sqrt{2\pi}^n} \cdot e^{-\frac{n}{2}} \\
s^2 &\stackrel{\Uparrow}{=} \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2
\end{aligned}$$

$$\begin{aligned}
\Lambda = \frac{L_0}{L} &= \left(\frac{s^2}{\sigma_0^2}\right)^{\frac{n}{2}} \cdot e^{-\frac{1}{2}\sum\left(\frac{x_i-\mu_0}{\sigma_0}\right)^2+\frac{n}{2}} \\
&= \left[\frac{1}{n}\sum\left(\frac{x_i-\bar{x}}{\sigma_0}\right)^2\right]^{\frac{n}{2}} \cdot e^{-\frac{1}{2}\sum\left(\frac{x_i-\mu_0}{\sigma_0}\right)^2+\frac{n}{2}} \\
-2\ln\Lambda &= -\not{2}\cdot\left\{\frac{n}{\not{2}}\ln\left[\frac{1}{n}\sum\left(\frac{x_i-\bar{x}}{\sigma_0}\right)^2\right] - \frac{1}{\not{2}}\sum\left(\frac{x_i-\mu_0}{\sigma_0}\right)^2 + \frac{n}{\not{2}}\right\} \\
&= n\cdot\ln n - n\cdot\ln \underbrace{\underbrace{\sum\left(\frac{x_i-\bar{x}}{\sigma_0}\right)^2}_{\chi^2_{n-1}} + \underbrace{\sum\left(\frac{x_i-\mu_0}{\sigma_0}\right)^2}_{\chi^2_n}}_{\text{dependientes}} - n
\end{aligned}$$

```
mu0 <- 0
sigma0 <- 1
n <- 100
simul <- replicate(100000,
{
  x <- rnorm(n)
  n*log(n) - n*log(sum(((x-mean(x))/sigma0)^2)) +
     sum(((x-mu0)/sigma0)^2) - n
})
a <- c(1,10,25,50,75,90,99)/100
round(rbind(quantile(simul,a), qchisq(a,2)), 3)
```

```
##         1%   10%   25%   50%   75%   90%  99%
## [1,] 0.02 0.212 0.577 1.398 2.805 4.652 9.21
## [2,] 0.02 0.211 0.575 1.386 2.773 4.605 9.21
```