Correction in **Approximations of upper and lower probabilities by measurable selections**

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There is an error in the proof of Theorem 8(2) in [5] [1]: the equality

$$P^*(A) = \sup\{P^*(K) : K \subseteq A \text{ compact}\} \ \forall A \in \beta_d \tag{1}$$

cannot be applied because [3, Lemma A3] is only applicable when X is a Polish space. In fact, Eq. (1) does not even hold in general when P is a precise probability measure on a separable metric space, because it need not be tight [1]. Below we give an alternative proof.

Theorem 1. *Let $(\Omega, \mathcal{A}, P)$ be a probability space, (X, d) a separable metric space and $\Gamma : \Omega \to \mathcal{P}(X)$ a compact random set. Then $P^*(A) = \max \mathcal{P}(\Gamma)(A)$ and $P_*(A) = \min \mathcal{P}(\Gamma)(A)$ $\forall A \in \beta_d$.*

Proof: For any closed subset C of X, the multivalued mapping

$$\begin{aligned} \Gamma_C :&\Omega \to \mathcal{P}(X) \\ &\omega \hookrightarrow \begin{cases} \Gamma(\omega) \cap C & \text{if } \omega \in \Gamma^*(C) \\ \Gamma(\omega) & \text{otherwise} \end{cases} \end{aligned}$$

is strongly measurable when Γ is. As a consequence, we can apply [2, Theorem III.8] to deduce that it has measurable selections, whence

$$P^*(C) = \max \mathcal{P}(\Gamma)(C)$$

for every closed set C. Now, by [4, Proposition 2], it holds that

$$P^*(A) = \sup\{P^*(C) : C \subseteq A \text{ closed}\} \ \forall A \in \beta_d.$$

Applying [5, Proposition 6], we deduce that $P^*(A) = \max \mathcal{P}(\Gamma)(A)$ for every $A \in \beta_d$. The result for P_* follows by conjugacy. ■

[1] We would like to thank Pedro Terán for pointing out this fact to us.

References

[1] Billingsley, P. (1968), *Convergence of probability measures.* Wiley, New York.

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[3] Castaldo, A., Maccheroni, F., and Marinacci, M. (2004), Random correspondences as bundles of random variables. *Sankhya*, 66(3): 409–427.

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[5] Miranda, E., Couso, I. and Gil, P. (2010). Approximations of upper and lower probabilities by measurable selections. *Information Sciences*, 180 (2010), 1407-1417.